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2 PATENT APPLICATION
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4 Docket No.: D349
5

6 Inventor(s): Gee L. Lui, Kuang Tsai
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8 Title: Gaussian Minimum Shift Keying (GMSK) Precoding Communication
9 Method
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11 SPECIFICATION
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13 Statement of Government Interest
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15 The invention was made with Government support under contract
16 No. F04701-93-C-0094 by the Department of the Air Force. The
17 Government has certain rights in the invention.
18

19 Field of the Invention
20

21 The invention relates to the field of continuous phase
22 modulation communications. More particularly, the present
23 invention relates to a Gaussian minimum shift keying communication
24 method for communicating a precoded digital data stream for
25 improved performance.
26
27

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Background of the Invention

4 Digital communication systems transmit data by various carrier
5 modulation techniques. The spectrum of a digital signal can be
6 controlled and made compact by envelope filtering or phase domain
7 filtering. Envelope filtering filters the baseband data stream
8 prior to upconversion to the carrier frequency and power
9 amplification. Envelope filtering for controlling out of band
10 power of the transmitted signal, can be found in many operational
11 communication systems. However, the power amplifier in the
12 transmitter must be linearized or backed off to prevent spectral
13 regrowth of the filtered signal. The more efficient phase domain
14 filtering approach controls the signal spectrum by frequency
15 modulating the filtered signal onto a carrier frequency to form a
16 continuous phase modulated (CPM) signal. The CPM signal has a
17 constant envelope so that the power amplifier can be operated at
18 maximum output power without affecting the spectrum of the filtered
19 signal.

21 Gaussian minimum shift keying (GMSK) is a form of continuous
22 phase modulation having compact spectral occupancy by choosing a
23 suitable bandwidth time product (BT) parameter in a Gaussian
24 filter. The constant envelope makes GMSK compatible with nonlinear
25 power amplifier operation without the concomitant spectral regrowth
26 associated with non-constant envelope signals. These attributes
27 render GMSK an attractive modulation scheme in all high throughput
28 frequency division multiple access satellite communication systems

1 where only a limited system bandwidth is available with the
2 transmitters operating at maximum power output efficiency.
3

4 Data bits are formatted, for example, by non-return to zero
5 (NRZ) formatting prior to Gaussian filtering, carrier modulation
6 and power amplification. The formatted data is transmitted within
7 data symbols of an M-ary alphabet of M possible data symbols. An
8 M-ary GMSK signal is defined by the complex envelope described in
9 terms of symbol energy E, symbol period T, carrier phase θ_c and
10 phase pulse $g(t)$ using a modulation index h and the equally
11 probable NRZ data symbols belong to an M-ary alphabet. The GMSK
12 phase pulse $g(t)$ originates from a frequency response $f(t)$ of the
13 Gaussian smoothing filter with a single-sided 3dB bandwidth B, time
14 truncated to a time interval of LT, where L is an integer. For
15 Gaussian filters with small BT products, the memory length L is
16 approximately an integer greater than or equal to 1/BT. The length
17 L is the number of elapsed symbol periods for the GMSK signal to
18 accrue a full phase change due to a single input symbol and hence
19 represents the memory of the GMSK signal. A GMSK signal with
20 memory L greater than one is termed a partial response GMSK signal.
21 The GMSK signal is communicated to a GMSK receiver subject to
22 interference and additive white Gaussian noise (AWGN).
23

24 An optimum GMSK receiver for an additive white Gaussian noise
25 channel demodulates the received signal by coherent demodulation
26 into estimated output data stream using a local carrier reference.
27 The receiver demodulates by filtering the received signal using a
28 bank of Laurent filters that filter the demodulated received signal

1 into a symbol sequence. A Viterbi decoder searches the symbol
2 sequence for the most probable transmitted data sequence as an
3 estimate of the original NRZ formatted data stream. A typical
4 coherent receiver for 2-ary GMSK signal is based on a pulse
5 amplitude modulation (PAM) representation of 2-ary CPM signals
6 using Laurent matched filters matched to the amplitude modulated
7 pulses in the PAM representation, and further employs the Viterbi
8 algorithm to optimally demodulate the symbol sequence. For 2-ary
9 GMSK transmitters with $BT=1/4$ and with a channel Bit Error Rate
10 (BER) of 0.01, or more, a GMSK receiver consisting of only two
11 matched Laurent filters and a 4-state Viterbi algorithm can nearly
12 achieve the performance of coherent binary phase shift Keying
13 (BPSK) signaling communications. Amplitude modulated pulses have
14 also been extended to PAM representation for 4-ary CPM signals.
15

16 In demodulating 2-ary and 4-ary GMSK signals using the Viterbi
17 algorithm, a differential decoder is necessary to resolve data bit
18 ambiguities while providing a degraded BER with respect to the
19 absolute phase demodulation. It is desirable to reduce the BER.
20 For noisy channels, the differential decoder yields poor bit error
21 rate performance. The Viterbi algorithm typically employs a
22 sliding window in the demodulation process where the width of the
23 sliding window represents the demodulation memory or delay. The
24 surviving state sequence $U_n = (S_n, S_{n-1}, \dots, S_{n-w})$ produced by the
25 sliding window Viterbi algorithm at any stage n depends on all the
26 demodulated symbols $\{d_k; 0 \leq k \leq n\}$ prior to that stage, where
27 $S_k = d_0 + d_1 + \dots + d_k$. The term W is the size of the sliding window that
28 is dictated by the memory length L of the underlying GMSK signal.

1 This intrinsic data dependency of the survivor state sequences U_n
2 disadvantageously requires a differential decoder operation in the
3 receiver when deciding on the actual demodulated symbol from
4 successive survivors of the Viterbi algorithm resulting in a
5 differential bit error rate (BER) degradation. These disadvantages
6 are solved or reduced by the present invention.

Summary of the Invention

9 An object of the invention is to improve the bit error rate
10 (BER) of continuous phase modulation communication systems.

12 Another object of the invention is to provide a precoding
13 method for continuous phase modulation communication systems.

15 Yet another object of the invention is to provide a precoding
16 method for a continuous phase modulation and coherent demodulation
17 communication system to reduce the BER while eliminating the need
18 for differential decoders in a receiver.

20 Still another object of the invention is to provide a
21 precoding method for a Gaussian minimum shift keying (GMSK)
22 continuous phase modulation and coherent demodulation communication
23 system to reduce the BER while eliminating the need for
24 differential decoders in a receiver.

26 The present invention is directed to a data precoding
27 algorithm implemented in a modulator of a transmitter to
28 substantially improve the resulting BER performance of the

1 continuous phase modulated (CPM) receivers, such as, Gaussian
2 minimum shift keying (GMSK) receivers without the use of
3 differential decoders while preserving the spectral occupancy of
4 the GMSK signals. The precoding algorithm offers performance
5 improvement for both 2-ary and 4-ary coherently demodulated GMSK
6 signals. The performance improvement afforded by the precoding
7 algorithm is up to 2.5 dB for coherent demodulation of 2-ary and 4-
8 ary GMSK signals. The precoding algorithm may be implemented using
9 a lookup table in the modulator circuitry without altering the
10 desired spectral occupancy of the non-precoded GMSK signals.

11

12 Precoding improves the BER performance for coherent
13 demodulators of the 2-ary and 4-ary GMSK signals implemented using
14 a pulse amplitude modulated signal subject to the Viterbi
15 algorithm. The precoding algorithm encodes the source NRZ data
16 symbols $\{d_n\}$ prior to the GMSK modulation so that the cumulative
17 phase of the precoded symbols $\{\alpha_n\}$ is identical to the absolute
18 phase of the source NRZ symbols at every stage of the Viterbi
19 algorithm, that is, $\pi h(\alpha_0 + \alpha_1 + \dots + \alpha_n) = \pi h d_n$, where h denotes
20 modulation index. In the Viterbi algorithm, this precoding process
21 produces a set of survivor sequences $\{U_n = (S_n, S_{n-1}, \dots, S_{n-w})\}$
22 satisfying the condition $S_k = d_k$ at every stage k , thus making the
23 differential decoder operation unnecessary. The precoded symbols
24 have the same statistics as the source symbols so that the transmit
25 spectrum of the GMSK signal is preserved. The BER is reduced using
26 the precoding method for GMSK signals with memory of L . The memory
27 L of a GMSK signal is related to the bandwidth time product BT of
28 the underlying Gaussian filter by $L=1/BT$, where B is the single-

1 sided 3dB filter bandwidth in hertz. Depending upon the channel
bit error rate required, the precoding method will render a signal
to noise ratio (SNR) improvement of up to 2.5 dB over the same
modem that demodulates GMSK signals without precoding. The
proposed precoding method may also be applied to other 2-ary and 4-
ary CPM signals. The only change required for these cases is to
modify the pulse amplitude modulation (PAM) filters of the
receiver. The precoding method encodes the source symbols $\{d_n\}$
prior to the GMSK modulation so that the resulting channel symbols
 $\{\alpha_n\}$ will render an optimal pseudo symbol sequence requiring no
differential decoding with improved bit error rates. These and
other advantages will become more apparent from the following
detailed description of the preferred embodiment.

Brief Description of the Drawings

17 Figure 1 is a block diagram of a precoded coherent Gaussian
18 minimum shift keying (GMSK) communication system.

20 Figure 2 is bit error rate (BER) performance graph for
21 precoded and non-precoded GMSK communication links using a Gaussian
22 filter having a bandwidth time product of $BT=1/3$.

28 | //

Detailed Description of the Preferred Embodiment

3 An embodiment of the invention is described with reference to
4 the figures using reference designations as shown in the figures.
5 Referring to Figure 1, a baseband representation of an M-ary
6 Gaussian minimum shift keying (GMSK) communication system is
7 simplistically shown for convenience to have ideal symbol timing
8 and carrier phase synchronization. The GMSK transmitter 10
9 comprises a data source 11, a data precoder 12, and a GMSK
10 modulator 14. The data source 11 continuously generates M-ary NRZ
11 data symbols d_n chosen from an M-ary alphabet set $\{\pm 1, \pm 3, \dots, \pm (M-1)\}$. These source symbols d_n are then precoded by the data precoder
12 12 into a precoded symbol sequence α_n that is in turn modulated by
13 the GMSK modulator 14. The GMSK modulator 14 includes a Gaussian
14 filter 13, a frequency modulator 15, and a frequency converter 16.
15 The Gaussian filter 13 is defined by a bandwidth time product (BT)
16 that may be, for example, 1/3, where B is the one sided 3dB
17 bandwidth in hertz of the Gaussian filter 13 and T is the data
18 symbol duration in seconds. For M-ary GMSK signals with $h=1/M$,
19 both main lobe bandwidth and sidelobe amplitude decrease with a
20 decreasing BT. The Gaussian filter 13 provides a Gaussian filter
21 output $G(t)$ expressed as an accumulative filter sum response of the
22 input sequence of precoded symbols α_n .

$$G(t) = \sum_n \alpha_n \cdot f(t - nT)$$

28 | //

1 The term $f(t)$ is the well-known GMSK frequency pulse that is a
2 function of the BT product and is essentially zero except over a
3 time interval of duration LT , where L is an integer representing
4 the memory of the Gaussian filter 13. The memory length L is
5 greater than or equal to one ($L \geq 1$). The frequency modulator 15
6 receives and modulates the Gaussian filter output $G(t)$ by a
7 predetermined modulation index h that may be, for example, $1/2$. In
8 general, lowering the modulation index h while keeping the BT
9 product constant will reduce spectral occupancy. Preferably, the
10 modulation index is set to $h=1/M$. The frequency modulator 15
11 transforms the Gaussian filter output $G(t)$ into a continuous phase
12 modulated baseband GMSK signal $z_b(t)$. The signal amplitude of
13 $\sqrt{2E/T}$ is taken as one.

14

15

16
$$z_b(t) = \exp \left\{ j\pi h \cdot \int_{-\infty}^t G(\tau) d\tau \right\} = \exp \left\{ j\pi h \sum_n \alpha_n \cdot \int_{-\infty}^t f(\tau - nT) d\tau \right\} \equiv \exp \left\{ j\pi h \sum_n \alpha_n \cdot g(t - nT) \right\}$$

17

18

19 The term $g(t)$, which is the integral of the GMSK frequency
20 pulse $f(t)$, is the well-known GMSK phase pulse.

22 The baseband GMSK signal $z_b(t)$ is then upconverted by the
23 converter 16 using a carrier reference 17 and then transmitted over
24 a communication channel 18 subject to additive white Gaussian noise
25 (AWGN) and potential interference 19. The transmitted GMSK signal,
26 along with noise and interference, is received by a corresponding
27 GMSK receiver 20 equipped with a frequency converter 21. The
28 converter 21 uses a locally generated carrier reference 22 to

1 downconvert the received RF signal into a baseband signal $Z_r(t)$.
2 The received baseband signal $Z_r(t)$ is then processed by a trellis
3 demodulator 24 to provide data estimates \hat{d}_n to a data sink 26. The
4 trellis demodulator 24 includes a filter bank 28, a sampler 30, and
5 a Viterbi decoder 32 implementing a Viterbi algorithm.

6

7 The received baseband signal $Z_r(t)$ is first filtered by the
8 filter bank 28 that consists of F matched filters, where F is at
9 most $Q=2^{L-1}$ for 2-ary GMSK signaling, and at most $P=3 \cdot [2^{L-1}]^2$ for 4-
10 ary GMSK signaling. The filters in the filter bank 28 are matched
11 to the Laurent amplitude-modulated pulses of the transmitted
12 baseband GMSK signal $Z_b(t)$, and may be implemented as a matched
13 filter bank or an integrate-and-dump type filter bank. The filter
14 bank 28 provides filtered signals $r_k(t)$ for $0 \leq k \leq F-1$, which are
15 sampled by the sampler 30 at every symbol time instants $t_n=nT$ to
16 produce discrete sample values $r_{k,n}$. These sample values are then
17 processed by the Viterbi decoder 32 to provide the data estimates \hat{d}_n
18 to the data sink 26. In order to produce reliable data estimates \hat{d}_n ,
19 the processing of the Viterbi decoder 32 must conform to the
20 precoding performed by the data precoder 12 on the data symbols d_n
21 at the transmitter 10. The number of matched filters used in the
22 filter bank 28 also affects the reliability of the data estimates \hat{d}_n .

23

24 For an N symbol long 2-ary data sequence $\{a_n; 0 \leq n \leq N-1\}$, the
25 baseband GMSK signal $Z_b(t)$ has a Laurent representation.

26

27

28

$$z_b(t) = \exp \left\{ j\pi h \cdot \sum_n \alpha_n \cdot g(t-nT) \right\} = \sum_{k=0}^{Q-1} \sum_{n=0}^{N-1} a_{k,n} \cdot h_k(t-nT)$$

The term $Q=2^{L-1}$ is the total number of 2-ary amplitude modulated pulses $\{h_k(t)\}$, and the term $\{a_{k,n}\}$ represents the pseudo-symbols relating to the 2-ary data sequence $\{\alpha_n\}$ through radix-2 digits $\{k_i\}$ for k defined by a summation over the index i with k_0 equal to zero.

$$k = \sum_{i=1}^{L-1} k_i \cdot 2^{i-1}$$

$$a_{k,n} = \exp \left\{ j\pi h \cdot \left[\sum_{m=0}^n \alpha_m - \sum_{i=0}^{L-1} k_i \cdot \alpha_{n-i} \right] \right\}$$

Each 2-ary amplitude modulated pulse $h_k(t)$ is related to the modulation index h and signal memory L through a generalized phase pulse $c(t)$.

$$h_k(t) = \prod_{i=0}^{L-1} c(t + iT - (1 - k_i)LT)$$

$$c(t) = \begin{cases} \sin[\pi h - \pi h g(|t|)] / \sin(\pi h), & |t| \leq LT \\ 0; & |t| \geq LT \end{cases}$$

///

1 The amplitude modulated pulse $h_k(t)$ is time limited to the
2 interval $[0, D_k T]$ where $D_k = \min_{0 < i < L} \{L(2 - k_i) - i\}$, for examples, $D_0 = L + 1$,
3 and $D_1 = L - 1$. The optimal trellis demodulator 24 for an N symbol
4 sequence $\{\alpha_n; 0 \leq n \leq N-1\}$ corrupted by additive white Gaussian
5 noise (AWGN) is one that maximizes 2^N correlation metrics for $0 \leq m \leq 2^N - 1$.
6

7

8

9 $\Lambda^{(m)} = \operatorname{Re}[\langle z_r(t), z_m(t) \rangle] = \operatorname{Re} \left[\int_{-\infty}^{\infty} z_r(t) \cdot z_m^*(t) dt \right]$

10

11

12 The term $z_m(t)$ denotes the baseband signal associated with the
13 m -th possible sequence, and the term $z_r(t)$ denotes the AWGN
14 corrupted received baseband signal. Expanding $z_m(t)$ in Laurent
15 representation, these correlation metrics can be expressed in terms
16 of the pseudo symbols $a_{k,n}^{(m)}$.

17

18 $\Lambda^{(m)} = \sum_{n=0}^{N-1} \lambda^{(m)}(n)$

19

20

21 $\lambda^{(m)}(n) \equiv \operatorname{Re} \left[\sum_{k=0}^{Q-1} r_{k,n} \cdot a_{k,n}^{(m)*} \right]$

22

23

24 $r_{k,n} \equiv \int_{-\infty}^{\infty} z_r(t) \cdot h_k(t - nT) dt = [z_r(t) * h_k(-t)]_{t=nT}$

25

26 The term $a_{k,n}^{(m)}$ denotes the pseudo symbols associated with the m -
27 th possible sequence. By supplying the sampled matched filter
28 output $r_{k,n}$ to the Viterbi decoder 32, which produces an optimal

1 pseudo symbol sequence $\{a_{0,n}; 0 \leq n \leq N-1\}$ that maximizes Λ , the
2 best estimate of the transmit sequence $\{\alpha_n\}$ can then be found. The
3 trellis demodulator is simplified by retaining the first F matched
4 filters $\{h_k(t); 0 \leq k \leq F-1\}$ in the λ equation, where F is often
5 confined to power of two due to the batch nature of the filter
6 duration $\{D_k\}$, e.g., $D_2=D_3=L-2$, $D_4=D_5=D_6=D_7=L-3$. For the case of
7 $F=2$, the first two 2-ary matched-filters and the corresponding λ
8 equation are explicitly given by matched filter equations h_0 and h_1 .

9

10
$$h_0(t) = \prod_{i=1}^L c(t-iT)$$

11

12

13
$$h_1(t) = h_0(t) \cdot \frac{c(t+T)}{c(t-(L-1)T)}$$

14

15

16

17
$$\lambda(n) \equiv \text{Re}[r_{0,n} \cdot a_{0,n}^* + r_{1,n} \cdot a_{1,n}^*]$$

18

19

20 The optimal pseudo symbol sequence produced by the Viterbi
21 decoder 32 at any stage n inevitably involves all the demodulated
22 symbols prior to that stage.

23

24
$$a_{0,n} = J^{(\alpha_0 + \alpha_1 + \dots + \alpha_n)}$$

25

26

27
$$a_{1,n} = J^{\alpha_n} \cdot a_{0,n-2}$$

1 The term $J=\exp(j\pi h)$ depends on the modulation index h , and is
2 $J=j$ for the case of $h=1/2$. This intrinsic data dependency requires
3 a differential decode operation when demodulating the actual data
4 symbol α_n .

5

6
$$\alpha_n = -j \cdot a_{0,n} \cdot a_{0,n-1}^*$$

7

8

9 This results in a differential BER degradation comparable to that
10 of DPSK as compared to BPSK.

11

12 The purpose of the data precoder 12 is to encode the source
13 symbols $\{d_n\}$ prior to the GMSK modulator 13 at the transmitter so
14 that the resulting precoded channel symbols $\{\alpha_n\}$ will render an
15 optimal pseudo symbol sequence $\{a_{0,n}\}$ requiring no differential
16 decoding in the receiver, thereby improving the data demodulation
17 performance. In mathematical terms, the data precoder 12 carries
18 out a data mapping.

19

20
$$\alpha_n = f(d_n, d_{n-1}, \dots, d_0)$$

21

22

23 The data mapping must provide a resulting expression for the pseudo
24 symbol $a_{0,n}$ involving only d_n and not $\{d_i; 0 \leq i < n\}$. There is no
25 known systematic routine that can be used to determine such a
26 mapping. Through repeated trial and error, and essentially by
27 chance, one such mapping for 2-ary GMSK signal with modulation
28 index $h=1/2$ has been found.

$$\alpha_n = d_n \cdot d_{n-1} = [d_n - d_{n-1} + 1]_{\text{mod}4} \quad (\alpha_0 = d_0)$$

This data mapping preserves the transmitted spectrum of the GMSK signal because the precoded symbols $\{\alpha_n\}$ are still equally probable as the source symbols $\{d_n\}$. The data mapping can be implemented in the data precoder 12 through the first 2-ary precoder lookup table.

First 2-ary Precoder Lookup Table		
d_n	d_{n-1}	$\alpha_n = [d_n - d_{n-1} + 1]_{\text{mod}4}$
-1	-1	+1
-1	+1	-1
+1	-1	-1
+1	+1	+1

The data mapping defined by the first 2-ary precoder lookup table results in an optimal pseudo symbol sequence produced by the Viterbi decoder 32.

$$a_{0,n} = J^n \cdot J^{d_n} = j^{n+1} \cdot d_n$$

$$a_{1,n} = J^{n-1} \cdot J^{d_n - d_{n-1} + d_{n-2}} = j^n \cdot d_n d_{n-1} d_{n-2}$$

1 With the decoding states defined as $S_n = (d_n, d_{n-1})$, a 2^2 -state
2 2^3 -branch Viterbi algorithm is sufficient for demodulating source
3 symbols $\{d_n\}$ when two 2-ary matched-filters are used in the filter
4 bank 28.

5
6 A second 2-ary data precoding mapping for 2-ary GMSK signal
7 with a modulation index $h=1/2$ has also been found.

8
9 $\alpha_n = (-1)^n \cdot d_n \cdot d_{n-1} = (-1)^n \cdot [d_n - d_{n-1} + 1]_{\text{mod}4} \quad (\alpha_0 = d_0)$

10
11
12 The second data precoding mapping also preserves the transmit
13 spectrum of the GMSK signal, and can be implemented in the data
14 precoder 12 through the second 2-ary precoder lookup table.

17 α Second 2-ary Precoder Lookup Table

d_n	d_{n-1}	$\alpha_n = [d_n - d_{n-1} + 1]_{\text{mod}4}$ n: even	$\alpha_n = [d_n - d_{n-1} + 1]_{\text{mod}4}$ n: odd
-1	-1	+1	-1
-1	+1	-1	+1
+1	-1	-1	+1
+1	+1	+1	-1

25
26 The data mapping defined by the second 2-ary precoder lookup
27 table results in an optimal pseudo symbol sequence produced by the
28 Viterbi decoder 32.

$$1$$
$$2$$
$$3$$
$$a_{0,n} = \begin{cases} j \cdot d_n; & n: \text{even} \\ d_n; & n: \text{odd} \end{cases}$$
$$4$$
$$5$$
$$6$$
$$7$$
$$8$$

$$9$$
$$10$$
$$a_{1,n} = \begin{cases} -d_n \cdot d_{n-1} \cdot d_{n-2}; & n: \text{even} \\ -j \cdot d_n \cdot d_{n-1} \cdot d_{n-2}; & n: \text{odd} \end{cases}$$
$$11$$

With the decoding states still defined as $s_n = (d_n, d_{n-1})$, a 2^2 -state 2^3 -branch Viterbi algorithm is again sufficient for demodulating source symbols $\{d_n\}$ when two 2-ary matched filters are used in the filter bank 28. The choice between the first and the second 2-ary data precoding mapping is arbitrary. Extensive simulations of the first and second mappings consistently yield identical demodulation performance.

Extending the 2-ary trellis demodulator to a 4-ary trellis demodulator is based on expressing every 4-ary symbol $\alpha_n \in \{\pm 1, \pm 3\}$ in terms of two 2-ary symbols $\alpha_n^{(0)}$ and $\alpha_n^{(1)}$.

$$25 \quad \alpha_n = \alpha_n^{(0)} + 2\alpha_n^{(1)}$$
$$26$$
$$27$$
$$28$$

1 This 4-ary α_n expression enables any 4-ary GMSK signal to be
2 expressed as a product of two 2-ary GMSK signals with $h^{(0)}=h$ and
3 $h^{(1)}=2h$ as the modulation indices, respectively.

4

5
$$\exp[j\pi h \sum_n \alpha_n g(t-nT)] = \exp[j\pi h^{(0)} \sum_n \alpha_n^{(0)} g(t-nT)] \times \exp[j\pi h^{(1)} \sum_n \alpha_n^{(1)} g(t-nT)]$$

6

7

8 Expressing each 2-ary signal constituent into Laurent
9 representation and combining the product, the baseband 4-ary GMSK
10 signal, for an N symbol long 4-ary data sequence $\{\alpha_n; 0 \leq n \leq N-1\}$,
11 takes an amplitude modulation pulse form. The signal amplitude of
12 $\sqrt{2E/T}$ is taken as one.

13

14
$$z_b(t) = \exp \left\{ j\pi h \cdot \sum_n \alpha_n \cdot g(t-nT) \right\} = \sum_{k=0}^{P-1} \sum_{n=0}^{N-1} b_{k,n} \cdot f_k(t-nT)$$

15

16

17

18

19 The term $P = 3Q^2$ is the total number of 4-ary amplitude
20 modulated pulses $\{f_k(t)\}$, and the terms $\{b_{k,n}\}$ are the 4-ary pseudo
21 symbols associated with the 4-ary data sequence $\{\alpha_n\}$. All the 4-ary
22 entities can be obtained from respective 2-ary counterparts by
23 following a systematic enumeration approach. The optimal 4-ary
24 trellis demodulator 24 is identical to that for 2-ary GMSK with the
25 following replacements for a λ equation and $r_{k,n}$ equation.

26

27
$$\lambda^{(m)}(n) \equiv \operatorname{Re} \left[\sum_{k=0}^{P-1} r_{k,n} \cdot b_{k,n}^{(m)*} \right]$$

28

$$r_{k,n} \equiv \int_{-\infty}^{\infty} z_r(t) \cdot f_k(t - nT) dt = [z_r(t) * f_k(-t)]_{t=nT}$$

The duration of the 4-ary amplitude modulated pulses $\{f_k(t)\}$ is also presented in batches, that is, $D_0=L+1$, $D_1=D_2=L$, $D_3=\dots=D_{11}=L-1$. The 4-ary trellis demodulator is simplified by retaining the first $F=1$, $F=3$ or $F=12$ matched filters in the filter bank 28. For the case of $F=3$, the 4-ary matched filters and the corresponding λ equation and pseudo symbols can be explicitly expressed.

$$\begin{aligned} f_0(t) &= h_0^{(0)}(t) \cdot h_0^{(1)}(t) \equiv h_0(t; h = h^{(0)}) \cdot h_0(t; h = h^{(1)}) \\ f_1(t) &= h_0^{(0)}(t+1) \cdot h_0^{(1)}(t) \equiv h_0(t+1; h = h^{(0)}) \cdot h_0(t; h = h^{(1)}) \\ f_2(t) &= h_0^{(0)}(t) \cdot h_0^{(1)}(t+1) \equiv h_0(t; h = h^{(0)}) \cdot h_0(t+1; h = h^{(1)}) \end{aligned}$$

$$\lambda(n) \equiv \text{Re}[r_{0,n} \cdot b_{0,n}^* + r_{1,n} \cdot b_{1,n}^* + r_{2,n} \cdot b_{2,n}^*]$$

$$\begin{aligned} b_{0,n} &= a_{0,n}^{(0)} \cdot a_{0,n}^{(1)} = J^{\alpha_0 + \alpha_1 + \dots + \alpha_n} \\ b_{1,n} &= a_{0,n-1}^{(0)} \cdot a_{0,n}^{(1)} = J^{2\alpha_n^{(1)} + \alpha_0 + \alpha_1 + \dots + \alpha_{n-1}} \\ b_{2,n} &= a_{0,n}^{(0)} \cdot a_{0,n-1}^{(1)} = J^{\alpha_n^{(0)} + \alpha_0 + \alpha_1 + \dots + \alpha_{n-1}} \end{aligned}$$

The term $J=(j\pi h)=(1+j)/\sqrt{2}$ for the case of $h=1/4$. The (0) and (1) terms indicate the modulation index being used for the 2-ary constituents, that is, $h^{(0)}=h$ or $h^{(1)}=2h$. As in the 2-ary GMSK case, the 4-ary pseudo-symbol $b_{0,n}$ at any stage n also involves

1 prior demodulated symbols, and must resort to the same differential
2 decode operation when demodulating the actual data symbol. Again,
3 through trial and error, two spectrum preserving data precoding
4 mappings have been found for the 4-ary GMSK signal with modulation
5 index $h=1/4$.

6

7
$$\alpha_n = [d_n - d_{n-1} + 1]_{\text{mod}8} \quad (\alpha_0 = d_0)$$

8

9

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11
$$\alpha_n = [d_n - d_{n-1} + 3]_{\text{mod}8} \quad (\alpha_0 = d_0)$$

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14 These precoding mappings can be implemented in the data
15 precoder 12 through the 4-ary precoder lookup table.

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4-ary Precoder Lookup Table			
d_n	d_{n-1}	$\alpha_n = [d_n - d_{n-1} + 1]_{mod 8}$	$\alpha_n = [d_n - d_{n-1} + 1]_{mod 8}$
-3	-3	+1	+3
-3	-1	-1	+1
-3	+1	-3	-1
-3	+3	+3	-3
-1	-3	+3	-3
-1	-1	+1	+3
-1	+1	-1	+1
-1	+3	-3	-1
+1	-3	-3	-1
+1	-1	+3	-3
+1	+1	+1	+3
+1	+3	-1	+1
+3	-3	-1	+1
+3	-1	-3	-1
+3	+1	+3	-3
+3	+3	+1	+3

The first precoding mapping $\alpha_n = [d_n - d_{n-1} + 1]_{mod 8}$ results in an optimal pseudo symbol sequences produced by the Viterbi decoder 32.

$$b_{0,n} = J^n \cdot J^{d_n}$$

$$b_{1,n} = J^{n-1} \cdot J^{d_{n-1} + 2\alpha_n^{(0)}}$$

$$b_{2,n} = J^{n-1} \cdot J^{d_{n-1} + \alpha_n^{(0)}}$$

1 The term J^n belongs to the set $\{\pm 1, \pm j, (\pm 1 \pm j)/\sqrt{2}\}$, and both
2 $\alpha_n^{(0)}$ and $\alpha_n^{(1)}$ are deterministic functions of d_n and d_{n-1} . Similarly,
3 the second precoding mapping $\alpha_n = [d_n - d_{n-1} + 3] \bmod 8$ results in an optimal
4 pseudo-symbol sequence produced by the Viterbi decoder 32.

5

$$6 \quad b_{0,n} = J^{3n} \cdot J^{d_n}$$
$$7 \quad b_{1,n} = J^{3n-3} \cdot J^{d_{n-1} + 2\alpha_n^{(1)}}$$
$$8 \quad b_{2,n} = J^{3n-3} \cdot J^{d_{n-1} + \alpha_n^{(0)}}$$

9

10

11 In both cases, with the decoding state defined as $S_n = (d_n)$, a
12 4¹-state 4²-branch Viterbi algorithm is sufficient for demodulating
13 the source symbols $\{d_n\}$ when three 4-ary matched-filters are used in
14 the filter bank 28.

15

16 Figure 2 quantified the performance improvement achieved
17 through the data precoding for both the 2-ary and 4-ary GMSK
18 signals with BT=1/3. Simulation data show that, depending on the
19 channel bit-error-rate of interest, a GMSK modem employing data
20 precoding will render a 0.5 dB to 2.5 dB signal-to-noise ratio
21 (SNR) enhancement over the same modem that employs no data
22 precoding.

23

24 The function of the precoder 12 in the GMSK transmitter 10 is
25 to precondition the symbol sequence α_k as an effective reverse
26 function of differential encoding that intrinsically results from
27 the GMSK modulation process. The precoding produces absolute phase
28 demodulation achieved within the GMSK receiver 20. This absolute
phase demodulation eliminates the need for differential decoding of

1 matched filters 28 while providing an improvement in signal
2 detection performance. The preferred precoding algorithms are
3 specific to M-ary CPM signals.

4

5 The present invention is directed to the precoding of a data
6 sequence into an encoded sequence of transmitted symbols, to avoid
7 differential decoding and for improving the BER using Laurent
8 filtering. In the preferred form, a precoder is applied to 2-ary
9 and 4-ary symbol sets used in a GMSK transmitter having a Gaussian
10 filter defines by respective BT products and frequency modulator
11 modulation indices. The preferred GMSK receivers included matched
12 filters 28, sampling 30, and Viterbi decoding 32. The general form
13 of the invention is a precoding method applicable to any M-ary
14 symbol set, BT product, modulation index, bank of match filters,
15 and Viterbi decoding algorithms. Those skilled in the art can make
16 enhancements, improvements and modifications to the invention, and
17 these enhancements, improvements and modifications may nonetheless
18 fall within the spirit and scope of the following claims.

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